

# Network Markets

Andre Veiga

Oxford, MPhil IO, MT 2015

# Outline

- 1 Intro
- 2 Multiplicity in Rohlfs (1974)
- 3 Simple Platform Model
- 4 Contingent pricing in Dybvig and Spatt (1983)
- 5 Intro to 2SM
- 6 Spence (1975) (via Weyl (2010))
- 7 Return to 2SM: Weyl (2010)
- 8 Competitive platforms: White and Weyl (2015)
- 9 Other Papers

# Motivation

- ▶ Networks: value of product depends on how many/who buys it
  - ▶  $\Rightarrow$  externalities between users
- ▶ Examples:
  - ▶ if many people have a phone, its more useful to have a phone
  - ▶ if many people use Word, it's more useful to have Word
  - ▶ if Brad Pitt wears blue shirt, average guy starts wearing blue
    - ▶ If I wear a blue shirt, average guy starts wearing red...
- ▶ Examples with 2 sides:
  - ▶ if a newspaper has lots of readers, this will attract advertisers...
    - ▶ which will repel readers...
    - ▶ which will repel advertisers
    - ▶ which will attract readers....
    - ▶ ...
  - ▶ if many shops accept a credit card, buyers will want to carry that card
    - ▶ shops will want to carry the card...

- ▶ We will refer to firms as “platforms”
- ▶ Sometimes connecting users is almost all the platform does
  - ▶ Facebook
  - ▶ Skype
  - ▶ Ebay
- ▶ Sometimes externalities are only part of the platform's value
  - ▶ software (compatibility)
  - ▶ nightclubs

# Several approaches to networks

- ▶ We will take a broad-scope “price theory” approach
  - ▶ looking at aggregate market measures like number of buyers
  - ▶ relate them to buyers types
  - ▶ characterize distortions
- ▶ Other approaches:
  - ▶ Graph theory: users are nodes in a graph (Jackson, Young, Teytelboym)
  - ▶ Detailed view of consumer interactions:
    - ▶ consumer bidding on Ebay
    - ▶ auction design by search engines
    - ▶ user searching & clicking online (White (2008))

# Goals

- ▶ math tools
  - ▶ differentiating fixed points
  - ▶ differentiating arbitrary integrals
- ▶ Overview of literature
  - ▶ where it is, where it's going
  - ▶ big gaps? limitations? **share your thoughts!**

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# Overview

- ▶ Big theme: multiplicity of equilibria in consumer game
- ▶ Setting: consumers joining a communications network
  - ▶ good to join if others join, not otherwise
  - ▶ focus on the consumer game
    - ▶ static platform

# Model

- ▶ One platform with fixed price  $p$
- ▶  $n$  individuals, indexed by  $i \in \{1, \dots, n\}$

$$x_i = \begin{cases} 0 & , \text{ if doesn't adopt} \\ 1 & , \text{ if adopts} \end{cases}$$

- ▶ Demand by Miss  $i$  is  $q_i(x_{-i}, p)$ , with  $x_{-i} \in \mathbb{R}^{n-1}$ 
  - ▶ decreasing in  $p$
  - ▶ increasing in every component of  $x_{-i}$ : positive network externalities
  - ▶ possibly micro-founded by utility  $U_i(x_{-i}, p)$  & outside option  $U_{i0}(x_{-i})$
  - ▶ allows for users to differ in preferences and values towards others
- ▶ Assumptions so far?

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  - ▶ allows for users to differ in preferences and values towards others
- ▶ Assumptions so far?
  - ▶ no congestion ( $q_i$  always increasing in  $q_{-i}$ )
  - ▶ everyone is desirable

## Equilibrium user sets

- ▶ Equilibrium user sets are solutions to the system of  $n$  equations:

$$q_i(x_{-i}, p) = 1 \Leftrightarrow U_i(x_{-i}, p) \geq U_{i0}(x_{-i}), \forall i$$

- ▶ Typically no unique solution  $\Leftrightarrow$  multiplicity
- ▶ Maximal eql: network is valuable  $\Leftrightarrow$  deviating (not joining) is not worthwhile
- ▶ Minimal eql: network is low value  $\Leftrightarrow$  deviating (joining) is not worthwhile
- ▶ bad news: platforms typically start with very few users
  - ▶ how do platforms leave the low-value equilibrium?

## More structure

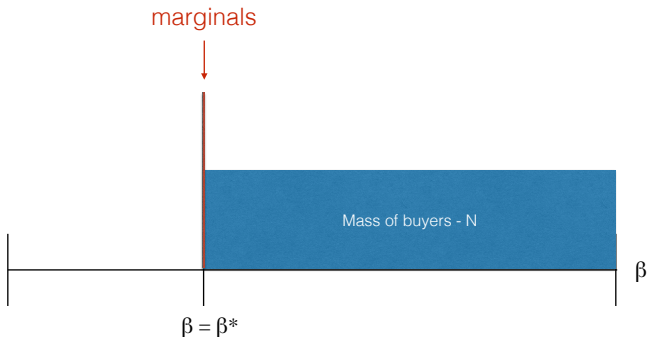
- ▶ Assume  $U_i = \frac{q}{n}\beta_i - p$  with  $q = \sum_i q_i$ 
  - ▶ additive utility
  - ▶ zero outside option
  - ▶ constant marginal utility for money
  - ▶ users care about only the total demand,  $q = \sum_i q_i$
  - ▶ join if  $U_i \geq 0$
  - ▶ types  $\beta_i > 0$  captures interaction benefits to user  $i$
- ▶ How good are these assumptions?

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  - ▶ types  $\beta_i > 0$  captures interaction benefits to user  $i$
- ▶ How good are these assumptions?
  - ▶ no intrinsic platform value  $\Rightarrow$  zero demand is an eql for any  $p \geq 0$
  - ▶ no congestion
  - ▶ homogeneous consumer value

## This buys us a simple demand structure

- Equilibrium joiners are the  $q$  people with the highest  $\beta_i$ :

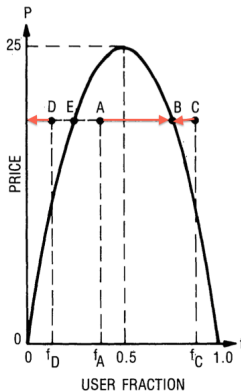


- There is still multiplicity! Multiple  $q$ 's can be equilibria for a given  $p$ .

## Even more structure: “uniform calling”

- ▶ We can order users  $\Rightarrow$  we can build a demand curve
- ▶ Continuum of users with mass  $n = 1$ , distribution  $v_i \sim \mathcal{U}[0, 100]$ 
  - ▶ if  $q$  users join, marginal user is  $\beta^* = 100(1 - q)$
  - ▶ marginal user has  $U_i = \beta^* q - p = 0 \Rightarrow p = 100q(1 - q)$ : a parabola
    - ▶ demand is not downward sloping in  $q \Rightarrow$  multiplicity

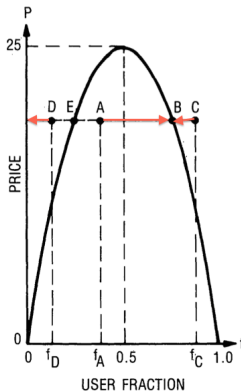
DEMAND CURVE FOR UNIFORM CALLING PATTERN





# Stability

DEMAND CURVE FOR UNIFORM  
CALLING PATTERN



- ▶ upward sloping demand  $\Rightarrow$  unstable equilibria
- ▶ downward sloping demand  $\Rightarrow$  stable equilibria

## One last issue: startup problem

- ▶ Viability: is there some equilibrium with positive profits?
- ▶ Can a start-up achieve a viable equilibrium from a small initial demand?
  - ▶ “chicken-and-egg” problem of Caillaud and Jullien (2003)
  - ▶ “failure to launch” of Evans and Schmalensee (2010)
- ▶ Rohlfs (1974) has a few thoughts:
  - ▶ half measures are worst, because then the whole effort might be lost
    - ▶ platforms business are risky?
  - ▶ best to give the service for free until the right user base is reached
    - ▶ Dhebar and Oren (1985): optimal dynamic path of prices
    - ▶ Veiga (2014):  $p|_{t=0} < 0$  optimal (cost of subsidy increases with  $q$ )

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- ▶ Now some real IO: platform pricing
  - ▶ focus on platform decision, not consumer game
  - ▶ exposition follows White (2012)
- ▶ Unit mass of consumers with utility  $u_i = v_i + \beta N - P$ 
  - ▶  $v_i \in \mathbb{R}$  heterogeneous “participation benefits”
    - ▶ reasonable for...Word? Facebook?
  - ▶ smooth PDF  $f(v)$
  - ▶  $\beta > 0$  homogeneous interaction benefits
  - ▶  $N \in [0, 1]$  is the expected measure of buyers
  - ▶ price  $P$
- ▶ Zero outside option
- ▶ Timing:
  - ▶ 1) platform chooses  $P$
  - ▶ 2) each consumer decides whether or not to join
- ▶ Assume expectation of  $N$  is correct in equilibrium
  - ▶ Fulfilled Expectation Cournot Equilibrium (Katz and Shapiro (1985))

# Solving

- ▶ Consumer joins if  $v_i \geq P - \beta N$
- ▶ Demand is

$$N = \mathcal{N}(P, N) = \int_{v^* = P - \beta N}^{\infty} f(v) dv$$

- ▶ Ignore multiplicity  $\Rightarrow \mathcal{N}$  is scalar-valued function
  - ▶ effectively, platform chooses  $N$
- ▶  $N = \mathcal{N}(P, N) = \mathcal{N}(P, \mathcal{N}(P, \mathcal{N}(P, \mathcal{N}(P, \dots))))$ 
  - ▶ fixed point

## Partial ( $\partial$ ) vs Total ( $d$ ) effects

- ▶ Profit is  $\pi = PN - c(N)$

- ▶ and  $N = \mathcal{N}(P, N)$

- ▶ FOC is

$$\frac{d\pi}{dP} = N + (P - c') \frac{dN}{dP} = 0$$

- ▶ To find  $\frac{dN}{dP}$ , differentiate  $N = \mathcal{N}(P, N)$ :

$$\underbrace{\frac{dN}{dP}}_{\text{total effect}} = \underbrace{\frac{\partial \mathcal{N}}{\partial P}}_{\text{direct partial effect}} + \underbrace{\frac{\partial \mathcal{N}}{\partial N} \frac{dN}{dP}}_{\text{indirect partial effect}} \Rightarrow \frac{dN}{dP} = \frac{\frac{\partial \mathcal{N}}{\partial P}}{1 - \frac{\partial \mathcal{N}}{\partial N}}$$

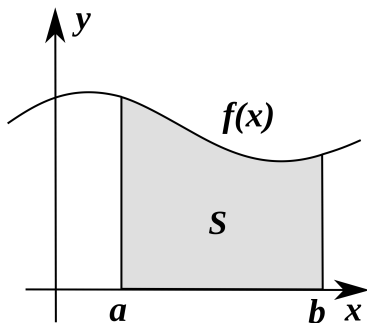
# Stability

$$\frac{dN}{dP} = \frac{\frac{\partial \mathcal{N}}{\partial P}}{1 - \frac{\partial \mathcal{N}}{\partial N}}$$

- ▶ Denominator captures the feedback/multiplier effect
- ▶ Stability if  $0 < \frac{\partial \mathcal{N}}{\partial N} < 1$  (Filistrucchi and Klein (2013))
  - ▶ network externalities are weak  $\Rightarrow$  system is not explosive
  - ▶ can interpret as  $\frac{1}{1 - \frac{\partial \mathcal{N}}{\partial N}} = 1 + \frac{\partial \mathcal{N}}{\partial N} + \frac{\partial \mathcal{N}^2}{\partial N} + \dots$
  - ▶  $\Rightarrow \mathcal{N}$  is a contraction  $\Rightarrow$  has a unique fixed point
  - ▶  $\Rightarrow \frac{dN}{dP} < 0$ : demand is overall downward sloping
  - ▶  $0 < \frac{\partial \mathcal{N}}{\partial N} < 1$  is joint condition on  $u_i$  and  $f(v)$

## Computing partial effects by the Leibniz Rule

- ▶ We need  $\frac{\partial \mathcal{N}}{\partial P}$  and  $\frac{\partial \mathcal{N}}{\partial N}$
- ▶ Differentiate  $\mathcal{N}(P, N)$  by Leibniz Rule:



$$\frac{dS}{dz} = \left[ \int_a^b \frac{df}{dz} dx \right] + \frac{db}{dz} f(b) - \frac{da}{dz} f(a)$$



## Partial effects

$$\mathcal{N}(P, N) = \int_{v^* = P - \beta N}^{\infty} f(v) dv$$

↓

$$\frac{\partial \mathcal{N}}{\partial P} = -\frac{\partial v^*}{\partial P} f(v^*) = -f(v^*) < 0$$

$$\frac{\partial \mathcal{N}}{\partial N} = -\frac{\partial v^*}{\partial N} f(v^*) = \beta f(v^*) > 0$$

- ▶ Signs are intuitive
- ▶ Stability if  $\frac{\partial \mathcal{N}}{\partial N} = \beta f(v^*) < 1$ : types are dispersed &  $\beta$  small

# Profit Maximization

- ▶ Profit is  $\pi = PN - c(N)$  and  $N = \mathcal{N}(P, N)$
- ▶ FOC is  $\frac{d\pi}{dP} = N + (P - c') \frac{dN}{dP} = 0$ :

$$P - c' = -\frac{N}{\frac{dN}{dP}} = -\frac{N}{\frac{\frac{\partial \mathcal{N}}{\partial P}}{1 - \frac{\partial \mathcal{N}}{\partial N}}} = \underbrace{\frac{N}{f(v^*)}}_{\text{Markup}} - \underbrace{\beta N}_{\text{Externality}}$$

- ▶  $\frac{N}{f(v^*)} > 0$  is the Cournot markup over marginal cost
  - ▶ it's the hazard rate of demand;  $f(v^*)$  is density of marginal users
- ▶  $-\beta N$  captures the effect of externalities:
  - ▶  $\beta > 0 \Rightarrow$  positive externalities, downward pressure on price
  - ▶ Why? Lowering price has two effects:
    - ▶ directly increases  $N$  (as usual)
    - ▶ extra feedback of  $N$  on itself, proportional to  $\beta$

# Welfare Maximization

- ▶ Welfare is  $W = -c(N) + \int_{v^*}^{\infty} (v + \beta N) f(v) dv$ , since  $u_i$  quasi-linear

$$\pi_{max} \Rightarrow P - c' = \frac{N}{f(v^*)} - \beta N$$

$$W_{max} \Rightarrow P - c' = 0 - \beta N < 0$$

- ▶ No markup
- ▶ price < marginal cost: Pigouvian subsidy to participation
  - ▶ externality from a marginal user to all infra-marginals is  $\beta N$
- ▶ Externality internalized by profit maximizer ( $\beta N$  in both FOCs)
  - ▶ not true if  $\beta$  heterogeneous (we'll see this later)

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# Ideas

- ▶ Externalities: actions are public goods
- ▶ Problems:
  - ▶ Multiplicity
  - ▶ Insufficient participation
- ▶ Solution: contingent payments

## Contingent payments

- ▶ Assume  $U_i = w_i(N) - p$ , with  $N = \sum_{j \neq i} x_j$ 
  - ▶ people care only about the total number of adopters
- ▶ Government wants to implement  $N^*$
- ▶ Gov commits to
  - ▶ charging adopters  $S(N, N^*)$
  - ▶ charging non-adopters  $T(N, N^*)$
- ▶ Intuition: choose  $S(N, N^*)$  such that
  - ▶  $N$  low  $\Rightarrow$  large payment  $\Rightarrow$  good to join
  - ▶  $N$  large  $\Rightarrow$  small payment  $\Rightarrow$  still good to join
  - ▶  $S(N, N^*)$  compensates, at each  $N$ , the  $N^*$ -th user
  - ▶ always obtain  $N^*$  in any equilibrium
  - ▶ if users are ranked (as before,  $N^*$  implies a unique set of buyers)
- ▶ It's similar to an insurance scheme: utility is guaranteed
  - ▶ cheap: no transfers in equilibrium (startup pricing in Rohlfs (1974) was costly)
  - ▶ Budget balances: raising  $S$  and  $T$  by  $\varepsilon$  preserves incentives

## Example

- ▶ Three possible consumers
  - ▶  $U_1 = 2N - p$
  - ▶  $U_2 = 8N - p$
  - ▶  $U_3 = N - P$
- ▶ Want to implement  $N^* = 2$ 
  - ▶ users 1 and 2 will join
  - ▶ User 2 will be the marginal user when  $N = 2$

$$S(N, N^* = 2) \begin{cases} P = 2 & , N = 1 \\ P = 4 & , N = 2 \\ P = 6 & , N = 3 \end{cases}$$

- ▶ Here,  $N^* = 2$  always corresponds to consumers 1 and 2 buying
  - ▶ this method works more generally (next lecture)

# Assumptions & Limitations

- ▶ Assume:
  - ▶ Gov need only know statistical distribution of preferences
    - ▶ no need to know who is who
    - ▶ Sakovics and Steiner (2012) use personalized subsidies
  - ▶ cannot use Groves mechanism
  - ▶ Gov can commit
  - ▶ payment can depend on the  $N$
  - ▶ Binary choices
- ▶ Limitations?



# Assumptions & Limitations

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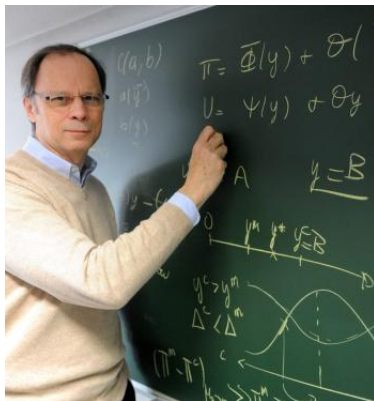
## ▶ Limitations?

- ▶ heterogeneous values? subsidy might attract the wrong users (Veiga and Weyl (Forthcoming))
- ▶ intensive margin? does it matter how much time people spend on Facebook?
- ▶ reversible decisions? switching costs?
- ▶ can platforms really commit?

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# Two sided markets



## ► Examples

- video games: gamers & developers
- newspapers: readers & advertisers
- straight dating websites: men & women
- job search engines: jobs & workers
- credit cards: shops & buyers

# Definition & Issues

## ► Definition

- multiple groups of users
- can be price discriminated
  - (or maybe quality-discriminated)
- demand depend on
  - price level
  - price structure
- externalities
  - across sides
  - maybe also within sides (as in the 1-sided models we saw)
  - examples?

## ► Issues:

- Does competition increase welfare? for which side?
- When is there collusion, predation, etc?

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## Weyl (2010) and Spence (1975)

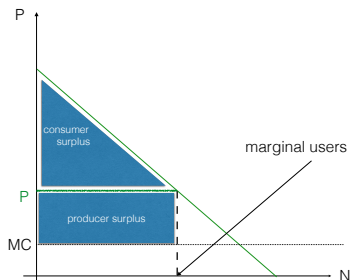
- ▶ Main idea of Weyl (2010):  $N$  is the quality of the platform
  - ▶ number of games = console quality for players
  - ▶ number of players = console quality for game developers
- ▶ Platform can choose quality on each side through price on other side
- ▶ Suppose 2 sides, A and B:
  - ▶  $P^A$  can be used to change  $N^A$ , which is quality towards side B
  - ▶  $P^A$  has two functions:
    - ▶ collect revenues from side A
    - ▶ set quality for side B
  - ▶ this was also true with 1SM we saw
    - ▶ changing price directly affected revenues
    - ▶ changed  $N$  (quality), which had a further effect on revenues
    - ▶ 2SM are not that different!
- ▶ Leverage Spence (1975) paper about quality-choosing monopoly...

## Spence (1975) profit and welfare

- Inverse demand is  $P(N, x)$ , cost per consumer is  $c(x)$

$$\pi(N, x) = N(P(N, x) - c(x))$$

$$W(N, x) = \int_0^N P(y, x) dy - c(x) N = N(\mathbb{E}[P(N, x)] - c(x))$$



- How does  $x$  change the demand curve?

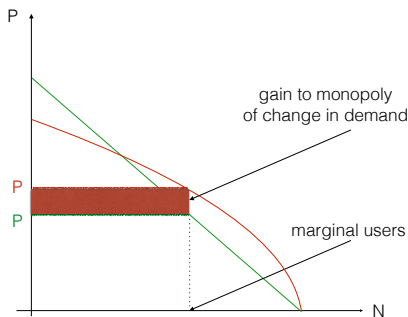
## Spence distortion

- $\pi = N(P(N, x) - c(x))$  and  $W = N(\mathbb{E}[P(N, x)] - c(x))$

$$\frac{\partial \pi}{\partial x} = 0 \Rightarrow c' = \frac{\partial P(N^*, x)}{\partial x}$$

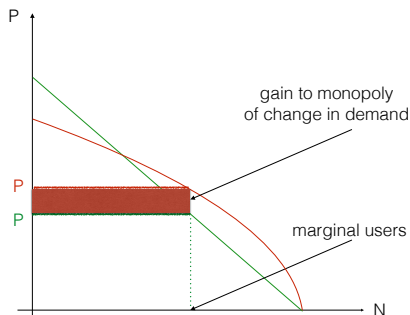
$$\frac{\partial W}{\partial x} = 0 \Rightarrow c' = \mathbb{E} \left[ \frac{\partial P(N, x)}{\partial x} \right]$$

- For profit, it only matters how  $x$  changes demand of marginals!





# Spence distortion



- ▶  $N$  is held fixed as  $x$  changes, so  $P$  implicitly adjusts
- ▶ When  $x$  increases, monopolist can:
  - ▶ keep the same  $N$  people
  - ▶ raise price to everyone ( $N$ )
  - ▶ price increase determine by preferences of marginals:  $\frac{\partial P(N^*, x)}{\partial x}$

# Examples

- ▶ City shops cater to tourists
- ▶ Film studios make movies that cater to kids
- ▶ Median voter theorem?
  - ▶ who are the marginal voters: undecided or abstaining?
- ▶ SIM cards are free, but customer service is often bad
- ▶ Hotelling location choices

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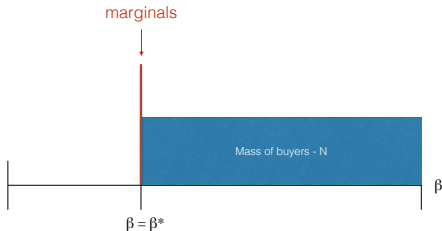
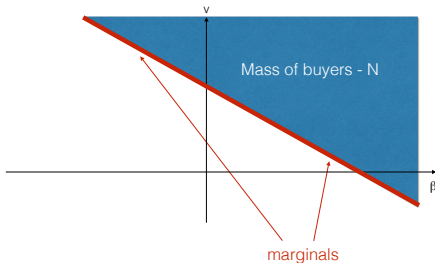
# Broad picture

- ▶ Generalization of several classical 2SM papers
  - ▶ Rochet and Tirole (2006)
  - ▶ Armstrong (2006)
- ▶  $N$  is quality (following Spence (1975))
- ▶ Insulating tariffs for uniqueness (following Dybvig and Spatt (1983))
- ▶ Multidimensional types
- ▶ Exposition follows White (2012)

# Model

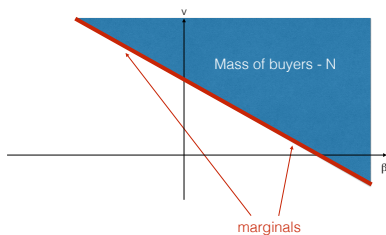
- ▶ two sides  $i \in \{A, B\}, j \neq i$
- ▶ platform chooses prices  $P^i$
- ▶ consumer utility  $u^i = v^i + \beta^i N^j - P^i$ 
  - ▶ types  $\theta^i = (v^i, \beta^i) \in \mathbb{R}^2$  has PDF  $f^i(\theta^i) > 0$  with full support
  - ▶  $N^i$  is number of consumers on side  $i$
  - ▶ only cross-side effects
  - ▶ outside option zero
  - ▶ what's new? 2 sides,  $\beta$  AND  $v$  both heterogeneous
- ▶ Buyers are  $\{v^i \geq P^i - \beta^i N^j\} = \{v^i \geq v^{i*}(\beta^i, P^i, N^j)\}$
- ▶ Marginals are  $\{v^i = v^{i*}(\beta^i, P^i, N^j)\}$ 
  - ▶ margin is defined by the function  $v^{i*}(\beta^i, P^i, N^j)$
  - ▶ there are several types on the margin, not just one

- ▶ Margin is defined by  $v^i = v^{i*}(\beta^i, P^i, N^j)$ : high  $v^i \Leftrightarrow$  low  $\beta^i$
- ▶ in 1D models, there is a unique type on the margin
  - ▶ here there are multiple



- Mass of buyers is

$$N^i = \mathcal{N}^i(P^i, N^j) = \int_{-\infty}^{\infty} \left[ \int_{v^{i*} = P^i - \beta^i N^j}^{\infty} f(v^i, \beta^i) dv^i \right] d\beta^i$$



## Total effect (d) of price

- ▶ Profit is

$$\pi = \sum_i N^i P^i - C(N^i, N^j)$$

- ▶ FOC includes total effect  $\frac{dN^i}{dP^i}$ .
- ▶ What are partial effects?
  - ▶ price directly affects demand:  $\frac{\partial \mathcal{N}^i}{\partial P^i}$
  - ▶ price changes  $N^i$ , this changes  $N^j$ , which feeds back to  $i$ :  $\frac{\partial \mathcal{N}^i}{\partial N^j}$

$$\frac{dN^i}{dP^i} = \underbrace{\frac{\partial \mathcal{N}^i}{\partial P^i}}_{\text{direct effect}} + \underbrace{\frac{\partial \mathcal{N}^i}{\partial N^j} \frac{\partial \mathcal{N}^j}{\partial P^i}}_{\text{indirect effect through } N^j}$$



## Total effect

- Compute the total effect from  $N^i = \mathcal{N}^i(P^i, N^j)$

$$\frac{dN^i}{dP^i} = \underbrace{\frac{\partial \mathcal{N}^i}{\partial P^i}}_{\text{direct effect}} + \underbrace{\frac{\partial \mathcal{N}^i}{\partial N^j} \frac{\partial \mathcal{N}^j}{\partial N^i} \frac{dN^j}{dP^i}}_{\text{indirect effect through } N^j} \Leftrightarrow \frac{dN^i}{dP^i} = \frac{\frac{\partial \mathcal{N}^i}{\partial P^i}}{1 - \frac{\partial \mathcal{N}^i}{\partial N^j} \frac{\partial \mathcal{N}^j}{\partial N^i}}$$

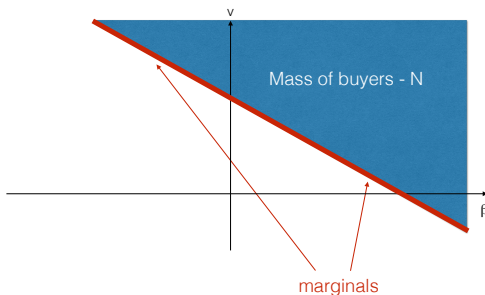
- $\frac{\partial \mathcal{N}^j}{\partial N^i}$  is symmetric to  $\frac{\partial \mathcal{N}^i}{\partial N^j}$
- Now stability requires  $\frac{\partial \mathcal{N}^i}{\partial N^j} \frac{\partial \mathcal{N}^j}{\partial N^i} < 1$ 
  - feedback depends on the interaction of the two sides
  - same interpretation as infinite feedback loop
  - overall downward sloping demand
- $\frac{\partial \mathcal{N}^i}{\partial N^j} \frac{\partial \mathcal{N}^j}{\partial N^i}$  have a symmetric form - we need only to compute one of them
- 2SM different, but quite similar

## Partial effect ( $\partial$ ) of price

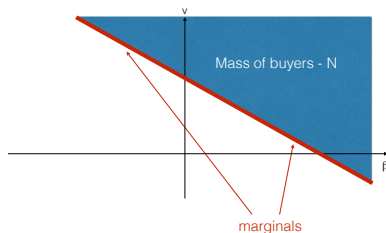
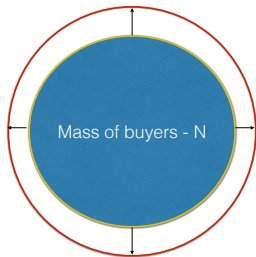
$$\mathcal{N}^i(P^i, N^j) = \int_{-\infty}^{\infty} \left[ \int_{v^{i*} = P^i - \beta^i N^j}^{\infty} f(v^i, \beta^i) dv^i \right] d\beta^i$$

$$\frac{\partial \mathcal{N}^i}{\partial P^i} = \int_{-\infty}^{\infty} \left[ -\frac{\partial v^{i*}}{\partial P^i} f(v^{i*}, \beta^i) \right] d\beta^i = - \int_{-\infty}^{\infty} f(v^{i*}, \beta^i) d\beta^i = -M^i$$

- ▶ This is the density of marginal buyers
  - ▶ before:  $N = \int_{v^*}^{\infty} f(v) dv$  and  $M = f(v^*)$
  - ▶ now:  $N$  is a double integral and  $M$  is a line integral



## Intuition/example: circle in 2D



- ▶ Area:  $N = \pi r^2$ . Then  $\frac{dN}{dr} = 2\pi r = M$  is the perimeter of circle
- ▶ Price is similar: shrinks set of buyers everywhere by the same amount because preferences are quasilinear

## Partial effect of quality ( $N^j$ )

$$N^i = \mathcal{N}^i(P^i, N^j) = \int_{-\infty}^{\infty} \left[ \int_{P^i - \beta^i N^j}^{\infty} f(v^i, \beta^i) dv^i \right] d\beta^i$$

$$\begin{aligned} \frac{\partial \mathcal{N}^i}{\partial N^j} &= \int_{-\infty}^{\infty} \left[ -\frac{\partial v^*}{\partial N^j} f(v^{i*}, \beta^i) \right] d\beta^i = \int_{-\infty}^{\infty} \beta^i f(v^{i*}, \beta^i) d\beta^i \\ &= M^i \frac{\int_{-\infty}^{\infty} \beta^i f(v^{i*}, \beta^i) d\beta^i}{\int_{-\infty}^{\infty} f(v^{i*}, \beta^i) d\beta^i} \\ &= M^i \mathbb{E}[\beta^i \mid v^i = v^{i*}] \end{aligned}$$

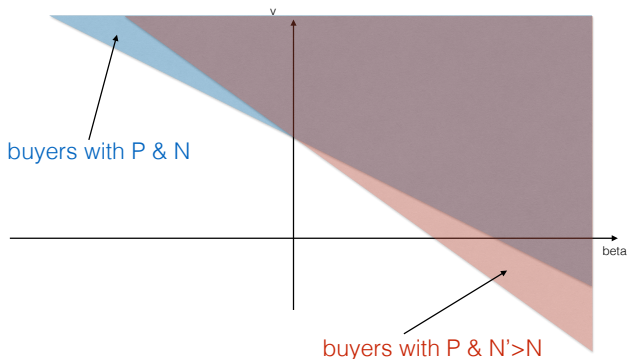
- ▶ The change in users on side  $i$ , when users on side  $j$  changes, depends on:
  - ▶ density of margin
  - ▶ marginal WTP for  $N^j$  among side- $i$  marginals
  - ▶ (marginals are the only ones who change their decision following a small change in  $N^j$ )

## Visual intuition

- Homogeneous prefs over  $P^i$ , but heterogeneous preferences over  $N^j$

$$\frac{\partial \mathcal{N}^i}{\partial P^i} = M^i \mathbb{E} \left[ \frac{\partial u^i}{\partial P^i} \mid v^i = v^{i*} \right] = -M^i$$

$$\frac{\partial \mathcal{N}^i}{\partial N^j} = M^i \mathbb{E} \left[ \frac{\partial u^i}{\partial N^j} \mid v^i = v^{i*} \right] = M^i \mathbb{E} [\beta^j \mid v^i = v^{i*}]$$



## FOCs

- ▶  $W = \sum_i \left\{ \int_{\beta^i} \int_{v^{i*}}^{\infty} (v^i + \beta^i N^j) f^i dv^i d\beta^i \right\} - C(N^i, N^j)$
- ▶  $\pi = \sum_i \{P^i N^i\} - C(N^i, N^j)$

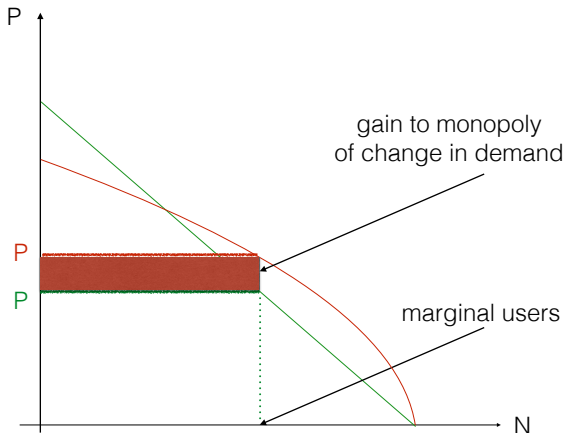
$$W_{max} \Rightarrow P^i - \frac{\partial C}{\partial N^i} = 0 - N^j \mathbb{E}[\beta^j \mid v^j \geq v^{j*}]$$

$$\pi_{max} \Rightarrow P^i - \frac{\partial C}{\partial N^i} = \underbrace{\frac{N^i}{M^i}}_{\text{markup}} - N^j \underbrace{\mathbb{E}[\beta^j \mid v^j = v^{j*}]}_{\text{Spence term}}$$

- ▶ The externalities are from side  $i$  to side  $j$ , hence  $N^j$  and  $\beta^j$
- ▶ Monopoly charges inefficient markup as before - Cournot distortion
- ▶ Spence distortion: Platform considers only marginal users!
  - ▶ when  $\beta$  was homogeneous, there was no Spence distortion

# Spence distortion

- ▶ Spence distortion: Platform considers marginal users
  - ▶ when  $N^i$  increases, platform captures from all  $N^j$  users, the surplus of marginal  $j$  users
  - ▶ absent when  $\beta$  was homogeneous ( $N^i$  simply shifts demand vertically)
  - ▶ not special to 2SM



# Spence distortion

- ▶ Sign of the Spence distortion depends on

$$\mathbb{E} [\beta^i \mid v^i = v^{i*}] \gtrless \mathbb{E} [\beta^i \mid v^i \geq v^{i*}]$$

- ▶ If  $\beta$  homogeneous, no distortion
- ▶ Spence can mitigate or exacerbate Cournot
- ▶ consequence of inability to price discriminate
- ▶ profit maximizing  $P^j$  might be negative if  $\mathbb{E} \left[ \frac{\partial u^i}{\partial N^j} \mid v^i = v^{i*} \right]$  large
  - ▶ would not occur in a 1-sided setting
  - ▶ regulation: zero price does not necessarily mean predation
  - ▶ lots of examples of zero pricing in 2SM: Gmail, Facebook, etc
  - ▶ negative prices might not work (users would create fake accounts)



## Price levels

$$\pi_{max} \Rightarrow P^i - \frac{\partial C}{\partial N^i} = \frac{N^i}{M^i} - N^j \mathbb{E}[\beta^j \mid v^j = v^{j*}]$$

- Which side is charged more? depends on

## Price levels

$$\pi_{max} \Rightarrow P^i - \frac{\partial C}{\partial N^i} = \frac{N^i}{M^i} - N^j \mathbb{E}[\beta^j \mid v^j = v^{j*}]$$

- ▶ Which side is charged more? depends on
  - ▶ elasticity of demand
  - ▶ how much you matter to the other side
    - ▶ as judged by their marginal users!
- ▶ If you opened a nightclub, would you charge more to women or men?

# Insulation

- ▶ platform can implement any  $(\hat{N}^i, \hat{N}^j)$  by committing to  $P^i(N^j)$ 
  - ▶ contingent prices, aka “insulating tariff”
  - ▶ “smooth” version of Dybvig and Spatt (1983)
- ▶ Then  $P^i(N^j)$  defined by the differential equation

$$\frac{dN^i}{dN^j} = 0 \Rightarrow \frac{\partial N^i}{\partial N^j} + \frac{\partial N^i}{\partial P^i} \frac{\partial P^i}{\partial N^j} = 0 \Rightarrow -\frac{\frac{\partial N^i}{\partial N^j}}{\frac{\partial N^i}{\partial P^i}} = \frac{\partial P^i}{\partial N^j}$$

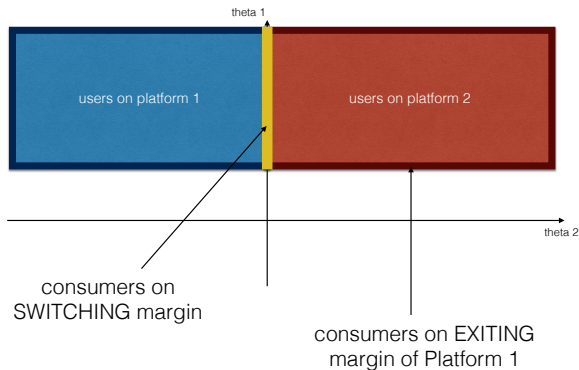
- ▶ Recall  $N^i = \mathcal{N}^i(P^i, N^j)$ . Boundary condition:  
 $\hat{N}^i = \mathcal{N}^i(P^i(\hat{N}^j), \hat{N}^j)$
- ▶ Intuition:
  - ▶ for any  $N^j$ , adjust  $P^i$  enough to obtain desired  $N^i$
  - ▶ requires  $\frac{\partial N^i}{\partial P^i} < 0$  for all  $N^j$  (true under regularity conditions on  $f^i$ )
  - ▶ prices might be negative
  - ▶ new: composition of buyers might change
  - ▶ monopolist only needs to insulate 1 side
- ▶ Same limitations as in Dybvig and Spatt (1983)

# Outline

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- 2 Multiplicity in Rohlfs (1974)
- 3 Simple Platform Model
- 4 Contingent pricing in Dybvig and Spatt (1983)
- 5 Intro to 2SM
- 6 Spence (1975) (via Weyl (2010))
- 7 Return to 2SM: Weyl (2010)
- 8 Competitive platforms: White and Weyl (2015)**
- 9 Other Papers

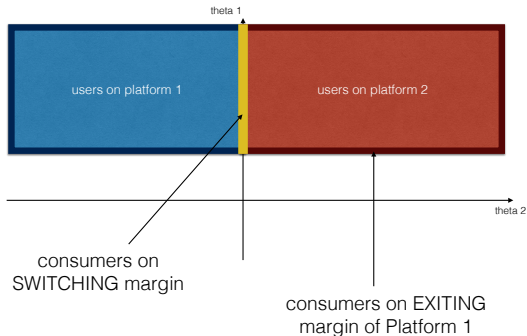
# Model

- ▶ Adding competition to Weyl (2010)
- ▶ Consider a market with 2 (1-sided) platforms, 1 and 2
  - ▶  $\theta_2$  is the Hotelling location
- ▶ There are two sets of “marginal users”
  - ▶ exiting margin: densities  $M_1^X$  and  $M_2^X$
  - ▶ common switching margin with density  $M^S$



## FOCs intuition

- ▶ Profit maximizer considers  $M = M^X + M^S$
- ▶ Welfare maximizer ignores S margin
  - ▶ S margin: same utility on either platform
  - ▶ increasing price  $\Rightarrow$  “lose” switching users
  - ▶  $\Rightarrow$  no loss in surplus (by envelope theorem)



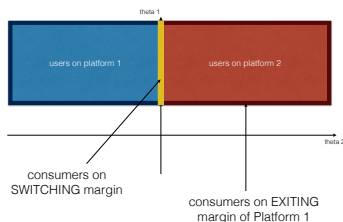
## FOCs math

$$W = \int_{\{Buyers^i(P^i, P^j)\}} (u^i) f(\theta^i) d\theta^i + \int_{\{Buyers^j(P^i, P^j)\}} (u^j) f(\theta^j) d\theta^j$$

- ▶ Must account for the 2 margins separately. Forgetting the externalities terms:

$$\frac{\partial W}{\partial P^i} = -M^{iX} - M^{iS} + M^{jS} = -M^{iX}$$

# Effect of Competition

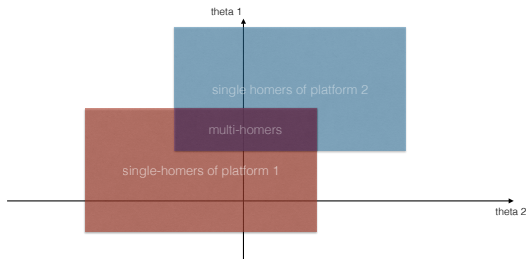


- ▶ Increasing competition  $\approx M^S$  increases
- ▶ Markup  $\frac{N}{M^S + M^X}$  decreases
- ▶ Competition increases weight of S increases, relative to X. What happens to Spence distortion?
  - ▶ if S users are representative  $\Rightarrow$  distortion decreases
  - ▶ if X users are representative  $\Rightarrow$  distortion increases
  - ▶ might be non-monotonic
  - ▶ perfect competition + symmetric equilibrium  $\Rightarrow$  everyone in S  $\Rightarrow$  no Spence distortion



# Multi-homing

- ▶ Users can “multi-home” (be on both platforms at once)
  - ▶ effectively, platform demands are independent
  - ▶  $\Rightarrow$  Firms are monopolies



- ▶ What if time spent on each network matters? multi-homers are less valuable than single-homers (Ambrus, Calvano and Reisinger (2014))

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▶ Katz and Shapiro (1985)

- ▶ static Cournot oligopoly with positive externalities
- ▶ firms choose whether their products are compatible
- ▶ large networks  $\Rightarrow$  oppose compatibility
- ▶ as a whole, firms have lower incentives for compatibility than society
- ▶ Fulfilled Expectation Cournot Equilibrium
  - ▶ consumer expectations about network size are realized in equilibrium

▶ Farrell and Saloner (1985)

- ▶ firms make sequential decision about whether to adopt a new standard or not
- ▶ payoff to adoption increases in number of adopters
- ▶ agents are better off moving earlier than later
- ▶ there can be excess inertia or excess momentum

- ▶ Biglaiser, Cremer and Veiga (2013)
  - ▶ explicit dynamics
  - ▶ consumers receive stochastic opportunities to switch
  - ▶ free riding incentive
  - ▶ there can be too much or too little switching
  - ▶ welfare loss from too much segregation
- ▶ Sakovics and Steiner (2012)
  - ▶ platform/gov knows consumers types and can solve coordination by giving personalized subsidies
- ▶ Jullien and Pavan (2013)
  - ▶ uniqueness in consumer game due to global games framework

# Thank you!

For questions:

[andre.veiga@economics.ox.ac.uk](mailto:andre.veiga@economics.ox.ac.uk)

[www.andreveiga.com](http://www.andreveiga.com)

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